The 38th Chinese Control Conference

Pre-conference Workshop 3
Distributed Optimization Over Networks

Abstract: In the distributed optimization over networks, nodes or agents cooperate to minimize an overall objective function that is the sum of per-node private objective functions. It finds a variety of applications, ranging from distributed estimation in sensor networks, distributed control of multirobot systems, synchronization of power systems, to machine learning using a cluster of computing units. Different from the centralized optimization, it involves local computations with communication among only a subset of nodes. In general, convergence rates of algorithms for distributed optimization depend not only on the number of nodes and the desired level of accuracy, but also on the structure and nature of communication networks.

Research on distributed optimization algorithms can be categorized by whether they are *synchronous* or *asynchronous*. The fundamental distinctness between them is that nodes in asynchronous algorithms do not wait for updates from others but simply compute updates by using its latest information available. Clearly, this involves a typical multi-timescale problem, which further brings forth new difficulties to the algorithm design. Despite significant advances have been made for synchronous algorithms, the design of practical asynchronous algorithms with theoretical performance guarantees proves notoriously challenging and has been somewhat an underdeveloped research area. As many experiments demonstrate advantages of asynchronous algorithms (e.g. fast convergence rates, easy to implement in practical systems and robust to uncertainties), the research focus has recently been shifted toward this area. In this workshop, we firstly present a brief overview of most recent work in the distributed optimization and propose an asynchronous (sub) gradient-push algorithm (AsySPA) for convex objective functions. The novel idea to address the asynchrony issue lies in the adaptive tuning of the update stepsize with local information. For strongly convex objective functions with Lipschitz continuous gradient, we further design an accelerated asynchronous algorithm with R-linear convergence.

Then, we talk about distributed optimization problems for multi-agent systems with physical dynamics where the output variables of networked agents are driven to simultaneously reach consensus on an optimal solution of the optimization problem. The additional constraints from the agent’s dynamics render the optimality achieving process quite different from the pure algorithmic framework. Particularly, the agent’s output variables cannot be altered in an arbitrary way but have

to obey the agent's physical dynamics. To solve this problem, we give two types of algorithms based on different information structures along with both state feedback and output feedback.

Finally, from the control point of view, we regard the network Laplacian operator as a pseudo projector and reformulate the problem of designing distributed optimization algorithms as a feedback-feedforward control problem. This allows us to design distributed algorithms and analyze their convergence by using the existing control tools such as dissipativity theory and integral quadratic constraints, which greatly simplifies the convergence analysis and provides potentials to design faster algorithms. Moreover, the control framework provides a relatively easy vehicle for addressing distributed *dynamic* optimization problems and we shall demonstrate it via application case studies.

In summary, the half-day pre-conference workshop mainly covers the following topics, all of which are essential to distributed optimization algorithms:

- The introduction of distributed optimization problems
- An overview of the-state-of-the-art distributed algorithms over networks
- The comparisons with *synchronous* and *asynchronous* algorithms
- The AsySPA for the distributed optimization problem
- The augmented graph approach for studying asynchronous algorithms
- The output distributed optimization for the higher-order agent's dynamics
- A control framework for distributed optimization
- The distributed dynamic optimization problem with application case studies
- Experiments on validating the proposed algorithms